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The unitary representations of the Diff R^N group

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Abstract. A new series of unitary representations of the general covariant group in N-dimensional real vector space (group Diff R^N) is constructed. The matrix elements of the finite transformations in the space of the principal series of unitary representations of the special linear group SL(N, R)($N > 3$) and in the space of all series of unitary representations of SL(2, R), SL(3, R) are found.

1. Introduction

In recent years there has been much interest in the unitary representations (UR) of the Diff R^N group† (see the review by Vershik *et al* 1975). It is important to construct the UR of Diff R^N because of their applicability to physical problems and the theory of UR of infinite Lie groups.

The finite-dimensional representations (FDR) of Diff R^N are well known. They are defined on the tensors $\Phi^{\alpha_1\ldots\alpha_r}_{\beta_1\ldots\beta_s}(x)(r, s = 1, 2, \ldots)$ and the pseudo-tensors $\Phi^{n;\alpha_1\ldots\alpha_r}_{\beta_1\ldots\beta_s}(x)$ of weight n (r, s = 1, 2, ...) (n is an integer) which under the Diff R^N transformation of coordinates $x_\mu \to x'_\mu(x)$ transform as

$$\Phi'^{\alpha_1\ldots\alpha_r}_{\beta_1\ldots\beta_s}(x') = \frac{\partial x'_{\alpha_1}}{\partial x_{\gamma_1}}\ldots\frac{\partial x'_{\alpha_r}}{\partial x_{\gamma_r}}\frac{\partial x_{\delta_1}}{\partial x'_{\beta_1}}\ldots\frac{\partial x_{\delta_s}}{\partial x'_{\beta_s}}\Phi^{\gamma_1\ldots\gamma_r}_{\delta_1\ldots\delta_s}(x) \tag{1}$$

and

$$\Phi'^{n;\alpha_1\ldots\alpha_r}_{\beta_1\ldots\beta_s}(x') = \left(\det\frac{\partial x'}{\partial x}\right)^n\frac{\partial x'_{\alpha_1}}{\partial x_{\gamma_1}}\ldots\frac{\partial x'_{\alpha_r}}{\partial x_{\gamma_2}}\frac{\partial x_{\delta_1}}{\partial x'_{\beta_1}}\ldots\frac{\partial x_{\delta_s}}{\partial x'_{\beta_s}}\Phi^{n;\gamma_1\ldots\gamma_r}_{\delta_1\ldots\delta_s}(x). \tag{2}$$

At any point $x_0 \in R^N$ the matrices

$$\left\{\frac{\partial x'_{\alpha_1}}{\partial x_{\gamma_1}}\ldots\frac{\partial x'_{\alpha_r}}{\partial x_{\gamma_2}}\frac{\partial x_{\delta_1}}{\partial x'_{\beta_1}}\ldots\frac{\partial x_{\delta_s}}{\partial x'_{\beta_s}}\right\}, \qquad \left\{\left(\det\frac{\partial x'}{\partial x}\right)^n\frac{\partial x'_{\alpha_1}}{\partial x_{\gamma_1}}\ldots\frac{\partial x'_{\alpha_r}}{\partial x_{\gamma_2}}\frac{\partial x_{\delta_1}}{\partial x'_{\beta_1}}\ldots\frac{\partial x_{\delta_s}}{\partial x'_{\beta_s}}\right\}$$

are matrices of the determined FDR of the linear group GL(N, R) for the element

$$g(x_0) = \left\{\frac{\partial x'_\mu}{\partial x_\nu}(x = x_0)\right\} \in \mathrm{GL}(N, R).$$

† More precisely the Diff R^N group is not a group in the present-day sense of the word (Singer and Sternberg 1965): it is a (Lie) pseudo-group which is defined in R^N by families of diffeomorphisms, closed under the operations of composition and inversion.

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In a recent paper (Borisov 1978, referred to hereafter as I) new series of UR for the Lie algebra of the Diff R^N group in the infinite-dimensional space of the UR of the SL(N, R) group were obtained. The present paper is a continuation of I. Here we construct a new series of UR of Diff R^N. They are defined in the infinite-dimensional space of the UR of the SL(N, R) group and are realised by infinite-dimensional matrices $T_{x'(x)}$. At any point $x_0 \in R^N$ the matrices $T_{x'(x)}$ are matrices of the determined UR of GL(N, R) for the element

$$g(x_0) = \left\{\frac{\partial x'_\mu}{\partial x_\nu}(x = x_0)\right\}.$$

2. The unitary representations of the Diff R^N group

2.1. The Iwasawa decomposition for the matrix $\Lambda(x'(x), x)$

First we assume all the conventions and notations of I. In what follows we refer to equation (. . .) of I as (I . . .). At every point R^N having coordinates x_μ we associate with the transformation (I1) the element $\Lambda(x'(x), x)$ of the group GL(N, R) defined by the matrix $\{\Lambda_{\mu\nu}(x'(x), x)\} = \{\partial x'_\mu(x)/\partial x_\nu\}$. Let us examine properties of this matrix, restricting ourselves to the transformations (I1) with $\det \Lambda(x'(x), x) > 0$. The matrix $\Lambda(x'(x), x)$ forms the representation (non-unitary) of Diff R^N, since the relation

$$\Lambda(x''(x'(x)), x'(x))\Lambda(x'(x), x) = \Lambda(x''(x'(x)), x) \tag{3}$$

is valid for any transformations $x_\mu \to x'_\mu(x)$, $x'_\mu(x) \to x''_\mu(x'(x))$ of Diff R^N. $\Lambda(x'(x), x)$ can be written as a product

$$\Lambda(x'(x), x) = d(\Lambda(x'(x), x))\Lambda_s(x'(x), x), \tag{4}$$

where the matrix $d(\Lambda(x'(x), x))$ is proportional to the unit matrix I: $d(\Lambda(x'(x), x)) = \det \Lambda(x'(x), x) \,.\, I$ and $\Lambda_s(x'(x), x) \in \mathrm{SL}(N, R)$. By the Iwasawa decomposition (Helgason 1962) $\Lambda_s(x'(x), x)$ can be written as a product

$$\Lambda_s(x'(x), x) = k(x)a(x)t(x), \tag{5}$$

where $k(x) \in \mathrm{SO}(N)$, $a(x) \in A$, $t(x) \in T$. In the following we will use two important properties of the Iwasawa decomposition: (i) the Iwasawa decomposition is unique; and (ii) the product of some element $k \in \mathrm{SO}(N)$ and the matrix $\Lambda_s(x'(x), x)$ can be written as

$$\Lambda_s(x'(x), x)k = k \,.\, \Lambda_s(x'(x), x)a(k, \Lambda_s(x'(x), x))t(x), \tag{6}$$

where $k \,.\, \Lambda_s(x'(x), x) \in \mathrm{SO}(N)$, $a(k, \Lambda_s(x'(x), x)) \in A$, $t(x) \in T$. The matrices $k \,.\, \Lambda_s^{-1}(x'(x), x)$ and $a(k, \Lambda_s^{-1}(x'(x), x))$ satisfy the relations

$$k \,.\, \Lambda_s^{-1}(x''(x'(x)), x) = k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda_s^{-1}(x'(x), x) \tag{7}$$

and

$$a(k, \Lambda_s^{-1}(x''(x'(x)), x)) = a(k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)), \Lambda_s^{-1}(x'(x), x))a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x))) \tag{8}$$

for any transformations $x_\mu \to x'_\mu(x)$, $x'_\mu(x) \to x''_\mu(x'(x))$ of the Diff R^N group. These relations follow from the associativity of a product of matrices, i.e.

$$(\Lambda_s^{-1}(x'(x), x)\Lambda_s^{-1}(x''(x'(x)), x'(x)))k = \Lambda_s^{-1}(x'(x), x)(\Lambda_s^{-1}(x''(x'(x)), x'(x))k),$$

and relation (3). Namely,

$$(\Lambda_s^{-1}(x'(x), x)\Lambda_s^{-1}(x''(x'(x)), x'(x)))k$$
$$= \Lambda_s^{-1}(x''(x'(x)), x)k$$
$$= k \,.\, \Lambda_s^{-1}(x''(x'(x)), x)a(k, \Lambda_s^{-1}(x''(x'(x)), x))t(x) \tag{9a}$$

and

$$\Lambda_s^{-1}(x'(x), x)(\Lambda_s^{-1}(x''(x'(x)), x'(x))k)$$
$$= \Lambda_s^{-1}(x'(x), x)k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x))a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x)))t_1(x)$$
$$= k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda_s^{-1}(x'(x), x)$$
$$\times a(k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)), \Lambda_s^{-1}(x'(x), x))t_2(x)$$
$$\times a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x)))t_1(x). \tag{9b}$$

Since T is an invariant subgroup in TA one has

$$t_2(x)a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x))) = a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x)))t_2'.$$

Therefore

$$\Lambda_s^{-1}(x'(x), x)(\Lambda_s^{-1}(x''(x'(x)), x'(x))k)$$
$$= k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda_s^{-1}(x'(x), x)a(k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)),$$
$$\times \Lambda_s^{-1}(x'(x), x))a(k, \Lambda_s^{-1}(x''(x'(x)), x'(x)))t_2'(x)t_1(x). \tag{10}$$

Then relations (7) and (8) follow from equations (9*a*, *b*) and (10) and the uniqueness of the Iwasawa decomposition. When the matrix $a(k, \Lambda_s^{-1}(x'(x), x))$ is written as $\exp(h(k, \Lambda_s^{-1}(x'(x), x)))$, relation (8) reads

$$h(k, \Lambda_s^{-1}(x''(x'(x)), x))$$
$$= h(k \,.\, \Lambda_s^{-1}(x''(x'(x)), x'(x)), \Lambda_s^{-1}(x'(x), x)) + h(k, \Lambda_s^{-1}(x''(x'(x)), x'(x))). \tag{11}$$

Using (4), (7) and (11) we obtain the decomposition

$$\Lambda(x'(x), x)k = k \,.\, \Lambda(x'(x), x)a(k, \Lambda(x'(x), x))t(x),$$
$$k \,.\, \Lambda(x'(x), x) \in \mathrm{SO}(N), \qquad a(k, \Lambda(x'(x), x)) \in A, \qquad t_2(x) \in T, \tag{12}$$

and relations

$$k \,.\, \Lambda^{-1}(x''(x'(x)), x) = k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda^{-1}(x'(x), x) \tag{13}$$

and

$$h(k, \Lambda^{-1}(x''(x'(x)), x)) = h(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)), \Lambda^{-1}(x'(x), x))$$
$$+ h(k, \Lambda^{-1}(x''(x'(x)), x'(x))), \tag{14}$$

where $a(k, \Lambda(x'(x), x)) = a(k, \Lambda_s(x'(x), x))d(\Lambda(x'(x), x))$. The group A has the generators $A_1, A_2, \ldots, A_N$ if $\det a \neq 1$ $(a \in A)$. Therefore

$$h(k, \Lambda^{-1}(x'(x), x)) = \sum_{i=1}^{N} t_i(k, \Lambda^{-1}(x'(x), x))A_i, \tag{15}$$

where $t_1, t_2, \ldots, t_N$ are the parameters of the group A.

2.2. The unitary representations of the Diff $R^N (N > 3)$ *group*

The UR of the principal series of the $SL(N, R)(N > 3)$ group was described in I. It is convenient to extend these representations to UR of the $GL(N, R)$ group using (I16), (I21) and the representation (I26) for the dilation generator. We obtain

$$\Psi'^{\omega}_{ij}(gx) = \sum_{\omega' i' j'} \int \mathrm{d}k\, t^{*\omega}_{ij}(k) t^{\omega'}_{i'j'}(k_{g^{-1}}) \left(\frac{\mathrm{d}k_{g^{-1}}}{\mathrm{d}k}\right)^{1/2} \exp(\alpha(h(k, g^{-1})))(\mathrm{d}_\omega\, \mathrm{d}_{\omega'})^{1/2} \Psi^{\omega'}_{i'j'}(x) \tag{16}$$

with

$$g^{-1}k = k_{g^{-1}} \exp(h(k, g^{-1}))t, \qquad k, k_{g^{-1}} \in SO(N), \qquad \exp(h(k, g^{-1})) \in A, \qquad t \in T,$$

where $\alpha(h(k, g^{-1}))$ is the linear function on the Lie algebra of the group A. The representation (16) is unitary with respect to the scalar product (I22) if $\alpha(h(k, g^{-1})) = -\alpha^*(h(k, g^{-1}))$.

Let us construct a family of representations of Diff $R^N (N > 3)$ parametrised by complex-valued functions on the Lie algebra of the group A. Firstly we introduce the auxiliary field $\Psi(k, x)$,

$$\Psi(k, x) = \sum_{\omega ij} \mathrm{d}^{1/2}_\omega\, t^\omega_{ij}(k) \Psi^\omega_{ij}(x), \tag{17}$$

and define the representation $T_{x'(x)}$ of the Diff R^N group by the prescription

$$\begin{aligned} \Psi'(k, x'(x)) &= T_{x'(x)} \Psi(k, x) \\ &= \left(\frac{\mathrm{d}k \,.\, \Lambda^{-1}(x'(x), x)}{\mathrm{d}k}\right)^{1/2} \exp(\alpha(h(k, \Lambda^{-1}(x'(x), x)))) \Psi(k \,.\, \Lambda^{-1}(x'(x), x), x). \end{aligned} \tag{18}$$

That $T_{x'(x)}$ is a representation of Diff R^N is straightforward:

$$\begin{aligned} T_{x''(x')} T_{x'(x)} \Psi(k, x) &= T_{x''(x')} \Psi'(k, x'(x)) \\ &= \left(\frac{\mathrm{d}k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x))}{\mathrm{d}k}\right)^{1/2} \exp(\alpha(h(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x))))) \\ &\quad \times \Psi'(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)), x'(x)) \\ &= \left(\frac{\mathrm{d}k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x))}{\mathrm{d}k}\right)^{1/2} \exp(\alpha(h(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x))))) \\ &\quad \times \left(\frac{\mathrm{d}k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda^{-1}(x'(x), x)}{\mathrm{d}k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x))}\right)^{1/2} \\ &\quad \times \exp(\alpha(h(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)), \Lambda^{-1}(x'(x), x)))) \\ &\quad \times \Psi(k \,.\, \Lambda^{-1}(x''(x'(x)), x'(x)) \,.\, \Lambda^{-1}(x'(x), x), x) \\ &= \left(\frac{\mathrm{d}k \,.\, \Lambda^{-1}(x''(x'(x)), x)}{\mathrm{d}k}\right)^{1/2} \exp(\alpha(h(k, \Lambda^{-1}(x''(x'(x)), x)))) \\ &\quad \times \Psi(k \,.\, \Lambda^{-1}(x''(x'(x)), x), x). \end{aligned} \tag{19}$$

The last step is due to the properties (13), (14) of the Iwasawa decomposition. Thus $T_{x''(x')} T_{x'(x)} = T_{x''(x'(x))}$. Let us denote $\alpha(A_i)(i = 1, 2, \ldots, N)$ by α_i. Then we write

$T_{x'(x)}$ more explicitly as

$$\Psi'(k, x') = T_{x'(x)}\Psi(k, x) = \left(\frac{dk\,.\,\Lambda^{-1}(x'(x), x)}{dk}\right)^{1/2}$$

$$\times \exp\left(\sum_{i=1}^{N} \alpha_i t_i(k, \Lambda^{-1}(x'(x), x))\right)\Psi(k\,.\,\Lambda^{-1}(x'(x), x), x).$$

The operator $T_{x'(x)}$ is realised on the fields $\Psi_{ij}^{\omega}(x)$ in the following form:

$$\Psi_{ij}^{\prime\omega}(x') = T_{x'(x)}\Psi_{ij}^{\omega}(x) = \int dk(d_\omega d_{\omega'})^{1/2} t_{ij}^{*\omega}(k)\left(\frac{dk\,.\,\Lambda^{-1}(x'(x), x)}{dk}\right)^{1/2}$$

$$\times \exp\left(\sum_{i=1}^{N} \alpha_i t_i(k, \Lambda^{-1}(x'(x), x))\right) t_{i'j'}^{\omega'}(k\,.\,\Lambda^{-1}(x'(x), x))\Psi_{i'j'}^{\omega'}(x). \tag{20}$$

The representation (20) is, in general, non-unitary with respect to the scalar product (I22) if α_i are the arbitrary complex numbers. Now suppose that $\alpha_i (i = 1, 2, \ldots, N)$ assume only pure imaginary values. It is easy to show that the representation (20) is then a UR of Diff R^N, i.e.

$$\sum_{\omega ij} \Psi_{ij}^{\prime *\omega}(x')\Psi_{ij}^{\prime\omega}(x') = \sum_{\omega ij} \Psi_{ij}^{*\omega}(x)\Psi_{ij}^{\omega}(x),$$

and has the form (16) for transformations of the GL(N, R) group.

In order to find the infinitesimal operators for the representation (20), we consider the infinitesimal transformation of coordinates, $x'_\mu = x_\mu + \epsilon f_\mu(x)(|\epsilon| \ll 1)$, of the Diff R^N group. Then the matrix elements $\Lambda_{\mu\nu}(x'(x), x)$ have the form

$$\Lambda_{\mu\nu}(x + \epsilon f(x), x) = \delta_{\mu\nu} + \epsilon\, \partial f_\mu/\partial x_\nu. \tag{21}$$

The infinitesimal generators GL(N, R) are the matrices $\tilde{F}_{\mu\nu}$ ($\mu, \nu = 1, 2, \ldots, N$), whose matrix elements are given by $(\tilde{F}_{\mu\nu})_{\alpha\beta} = -i\delta_{\mu\alpha}\delta_{\nu\beta}$. Therefore

$$\Lambda(x + \epsilon f(x), x) = I + i\epsilon(\partial f_\mu/\partial x_\nu)\tilde{F}_{\mu\nu}. \tag{22}$$

From equations (20) and (22) we obtain the UR for the Lie algebra of Diff R^N ($N > 3$) constructed in I (see I44a)):

$$\Psi_{ij}^{\prime\omega}(x + \epsilon f(x)) = T_{x+\epsilon f(x)}\Psi_{ij}^{\omega}(x) = \Psi_{ij}^{\omega}(x) + i\epsilon(\partial f_\mu/\partial x_\nu)(\tilde{F}_{\mu\nu})_{\omega ij,\omega' i'j'}\Psi_{i'j'}^{\omega'}(x)$$

$$= (I + \epsilon \bar{T}_f)_{\omega ij,\omega' i'j'}\Psi_{i'j'}^{\omega\prime}(x). \tag{23}$$

2.3. *The unitary representations of* Diff R^2 *and* Diff R^3

We proceed by analogy with the construction of the UR of Diff $R^N (N > 3)$ and try to define the UR of Diff R^2 and Diff R^3 in the space of all series of the UR of SL(2, R) and SL(3, R) respectively. It is convenient to extend the UR of SL(2, R) and SL(3, R) which are described in I to a UR of GL(2, R) and GL(3, R) respectively. From (I17), (I19) and (I21), (I26) we find the realisation of UR of GL(2, R) and GL(3, R) in the following form. For three series of the UR of GL(2, R)

$$\Psi'_m(gx) = (T(g))_{mn}\Psi_n(x)$$

$$= \int dk\, f_m^*(k)(a_{11}(k, g^{-1}))^{(id-s+1)/2}(a_{22}(k, g^{-1}))^{(id+s-1)/2} f_n(k_{g^{-1}})\Psi_n(x), \tag{24}$$

with

$$g^{-1}k = k_{g^{-1}}a(k, g^{-1})t, \qquad k, k_{g^{-1}} \in \mathrm{SO}(2), \qquad g \in \mathrm{GL}(2, R), \qquad \det g > 0.$$

For three series of the UR of GL(3, R)

$$\Psi'_{lnm}(gx) = (T(g))_{lnm,l'n'm'}\Psi_{l'n'm'}(x)$$

$$= \int \mathrm{d}k\, \tau^{*l}_{nm}(k)(a_{11}(k, g))^{(2\mu-\lambda+id)/3}(a_{22}(k, g))^{(id-\mu+2\lambda)/3}$$

$$\times (a_{33}(k, g))^{(-\mu-\lambda+id)/3}[(2l+1)(2l'+1)]^{1/2}\tau^{l'}_{n'm'}(k_g)\Psi^{l'}_{n'm'}(x), \tag{25}$$

with

$$kg = ta(k, g)k_g, \qquad g \in \mathrm{GL}(3, R), \qquad \det g > 0,$$

$$k, k_g \in \mathrm{SO}(3), \qquad a(k, g) \in A, \qquad t \in T.$$

Replacing the matrix elements $g_{\mu\nu}(\mu, \nu = 1, 2)$ by $\Lambda_{\mu\nu}(x'(x), x)$ $(x \in R^2)$ in the infinite matrix $\{(T(g))_{mn}\}(g \in \mathrm{GL}(2, R))$ we obtain the infinite matrix $\{(T_{x'(x)})_{mn}\}$ as follows:

$$(T_{x'(x)})_{mn} = \int \mathrm{d}k\, f^*_m(k)(a_{11}(k, \Lambda^{-1}(x'(x), x)))^{(id-s+1)/2}$$

$$\times (a_{22}(k, \Lambda^{-1}(x'(x), x)))^{(id+s-1)/2}f_n(k\,.\,\Lambda^{-1}(x'(x), x)). \tag{26}$$

It is evident that

$$\Psi'_m(x') = (T_{x'(x)})_{mn}\Psi_n(x) \tag{27}$$

is a UR (with respect to the scalar product (I18), (I22)) of Diff R^2. The UR of Diff R^3 is constructed in a similar way:

$$\Psi'_{lnm}(x') = \int \mathrm{d}k\, \tau^{*l}_{nm}(k)(a_{11}(k, \Lambda(x'(x), x)))^{(2\mu-\lambda+id)/3}$$

$$\times (a_{22}(k, \Lambda(x'(x), x)))^{(id-\mu+2\lambda)/3}(a_{33}(k, \Lambda(x'(x), x)))^{(-\mu-\lambda+id)/3}$$

$$\times [(2l+1)(2l'+1)]^{1/2}\tau^{l'}_{n'm'}(k\,.\,\Lambda(x'(x), x))\Psi^{l'}_{n'm'}(x), \tag{28}$$

with

$$k\Lambda(x'(x), x) = ta(k, \Lambda(x'(x), x))k\,.\,\Lambda(x'(x), x),$$

$$k, k\,.\,\Lambda(x'(x), x) \in \mathrm{SO}(3), \qquad a(k, \Lambda(x'(x), x)) \in A.$$

It is evident that the representation (28) is unitary with respect to the scalar product (I20), (I22) if the parameters d, μ, λ have values given in I.

3. Summary

In this work we have constructed the unitary representations of the Diff R^N group in the space of such UR of SL(N, R) which have realisation in the space of functions on SO(N). We have used a method which is closely connected with the method of induced representations (Warner 1972) in the representation theory of semi-simple finite Lie groups; apart from the theoretical interest, it is an advantage in practical calculations.

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References

Borisov A B 1978 *J. Phys. A: Math. Gen.* **11** 1057
Helgason S 1962 *Differential Geometry and Symmetric Spaces* (New York: Academic) ch 6
Singer I M and Sternberg S 1965 *J. Analyse Math.* **15** 3
Vershik A M, Gelfand I M and Graev M I 1975 *Usp. Mat. Nauk.* **30** (No. 6) 3
Warner G 1972 *Harmonic Analysis on Semi-Simple Groups* vol 1 (Berlin: Springer) ch 5